# Time Series Econometrics: Some Basic Concepts Non-Stationary Variables

### Outline

- Stationary: What & Why?
- 1. What & Why
- 2. How to Test Stationarity
- 3. How to overcome Non-Stationarity
- **Spurious Regression**
- Cointegration and Error Correction Model

### Stationary

- Data: Stationary and Non Stationary
- Stationary: The mean, variance and covariance is constant and time invariant  $\circ$  E.g. let's  $\mathbf{Y}_t$  be a stochastic process, then;

  - Covariance:  $\gamma_k = E[(Y_t \mu)(Y_{t+k} \mu)]$  .....(3)
    - Where  $\gamma_k$ , the covariance (or auto-covariance) at lag k,
    - If k = 0, we obtain  $\gamma_0$ , which is simply the variance of Y (=  $\sigma^2$ ); if k = 1,  $\gamma_1$  is the covariance between two adjacent values of Y

#### Non-stationary

- The mean and variance is time varying or not constant:
- ✓ Random walk without drift (increasing in variance  $\mathbf{Y}_t = \mathbf{Y}_{t-1} + \mathbf{u}_t$ )
- ✓ Random walk with drift (variance and mean is not constant)

$$\mathbf{Y}_{t} = \mathbf{\delta} + \mathbf{Y}_{t-1} + \mathbf{u}_{t}$$

 $\checkmark$  Random walk with drift around a stochastic trend (Y  $_t$  =  $\beta_1$  +  $\beta_2 t$  + Yt\_1  $_+$  ut)

# Why?

- If the data is not stationary, than the OLS estimation is bias because the mean and the variance is time varying and not constant
- Unable to make prediction about the relationship among dependent and independent variables
- ➢Unable to perform forecasting for short term and also for the long term.

### Unit Root Test – Three Types

- By graphical analysis Plot the graph whether to see the trend has change or has a constant variation
- Autocorrelation function (ACF) Box–Pierce Q statistic
- Unit Root Test
- i. Augmented Dickey Fuller Test (ADF)

#### How to detect stationary and non stationary

- In practice we face two important questions:
  - How do we find out if a given time series is stationary or not?
  - Is there a way that it can be made stationary?
- Prominently discussed tests in the literature are:
  - Graphical Analysis
  - The Unit Root Test

### Graphical approach LGDP and LPDI



# Autocorrelation Function (ACF)-significant test

- Q test the standard error test
- Hypothesis
- $H_0: \rho_k = 0$  (stationary)
- $H_a: \rho_k \neq 0$  (nonstationary)
- K=lag
- Critical value  $\alpha = 5\%$

• 
$$-1.96(se) < \rho_k < 1.96(se) = -1.96\left(\sqrt{\frac{1}{n}}\right) < \rho_k < 1.96\left(\sqrt{\frac{1}{n}}\right)$$
.  $n = high observations$ 

• If the statistic  $Q < \chi^2(\alpha)$  do not reject H null, mean the time series is stationary

#### Autocorrelation Function (ACF) for LGDP

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		2	0.905	-0.056	156.92	0.000	
		3	0.890	-0.029	227 50	0.000	
		4	0.851	-0.030	292.85	0.000	
		5	0.812	-0.016	353.15	0.000	
		6	0.773	-0.024	408.53	0.000	
		7	0.735	-0.017	459.19	0.000	
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		14	0.492	-0.013	707.34	0.000	
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		16	0.420	-0.023	747.88	0.000	
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		18	0.342	0.009	776.28	0.000	
	! ! !	19	0.307	0.004	786.76	0.000	
	! ! !	20	0.2/4	0.004	795.22	0.000	
		21	0.243	-0.001	801.98	0.000	
		22	0.211	-0.042	807.10	0.000	
		23	0.170	-0.034	010.92	0.000	
	:4: /	24	0.140	-0.025	015.00	0.000	
		25	0.117	0.000	016 1/	0.000	
		20	0.063	0.002	916.65	0.000	
		28	0.004	-0.035	916.84	0.000	
		20	0.030	0.035	040.00	0.000	¥

For LGDP, the value of the *Q* statistic up to lag 36 is about 821.51 The probability of obtaining such a *Q* value under the null hypothesis that the sum of 36 squared estimated autocorrelation coefficients is zero is practically zero show the that LGDP is nonstationary

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#### ACF for LPDI

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		35	-0.108	0.015	863.45	0.000	
· 🖣 ·	1 1 1 1	36	-0.126	-0.021	865.84	0.000	

 For LPDI, the value of the Q statistic up to lag 36 is about 865.84 The probability of obtaining such a Q value under the null hypothesis that the sum of 36 squared estimated autocorrelation coefficients is zero is practically zero – show the that LPDI is nonstationary

# Perform Unit Root Test to Test For Stationarity: Augmented Dickey Fuller Test

- H null: Series has unit root (meaning series is non-stationarity)
- Series assumptions (your decision-Augmented Dickey Fuller Approach):
- 1. Constant (i.e intercept) (  $\Delta Y_{t-1} = \alpha + \delta Y_{t-1} + \varepsilon_t$ )
- 2. Constant and trend  $(\Delta Y_{t-1} = \alpha + \alpha_2 t + Y_{t-1} + \varepsilon_t)$
- 3. None  $(\Delta Y_{t-1} = Y_{t-1} + \varepsilon_t)$

$$\Delta Y_t = \alpha + \emptyset t + \gamma^c Y_{t-1} + \sum_{i=1}^n \emptyset \Delta Y_{t-i} + \varepsilon$$

• If H null is accepted (i.e series has a unit root), it must be differenced to see if stationarity is achieved after 1<sup>st</sup> differencing

Visualize to determine the your decision:



 The LGDP and LPDI seems o be drifting or a drift - but not a deterministic trends

#### The ADF test – at level for LGDP

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### The ADF test – First Difference for LGDP

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#### ADF test – LPDI at Level

Augmented Dickey-Fuller Unit Root Test on LPDI	Series: LPDI Workfile: TIME SERIES PART 1::Untitled\
Unit Root Test	View Proc Object Properties Print Name Freeze Sample Genr Sheet Graph Stats Augmented Dickey-Fuller Unit Root Test on LPDI
Test type Augmented Dickey-Fuller	Null Hypothesis: LPDI has a unit root       A         Exogenous: Constant       Lag Length: 0 (Automatic - based on SIC, maxlag=11)         t-Statistic       Prob.*
Test for unit root in       Lag length         O Level       Automatic selection:         Schwarz Info Criterion       Schwarz Info Criterion	Augmented Dickey-Fuller test statistic         -1.416752         0.5703           Test critical values:         1% level         -3.511262           5% level         -2.896779           10% level         -2.585626   *MacKinnon (1996) one-sided p-values.
Include in test equation       Maximum lags: 11         Intercept       Trend and intercept         None       User specified: 2	Augmented Dickey-Fuller Test Equation Dependent Variable: D(LPDI) Method: Least Squares Date: 07/11/21 Time: 20:20 Sample (adjusted): 1971Q2 1991Q4 Included observations: 83 after adjustments
OK Cancel	Variable         Coefficient         Std. Error         t-Statistic         Prob.           LPDI(-1)         -0.010311         0.007278         -1.416752         0.1604           C         0.088285         0.057762         1.528433         0.1303

## The ADF test – First Difference for LPDI

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	10% level		-2.585861		
*MacKinnon (1996) on	e-sided p-values				
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Variable	Coefficient	Std. Error	r t-Statistic	Prob.	
D(LPDI(-1)) C	-1.053159 0.006752	0.111602 0.001399	2 -9.436743 0 4.827515	0.0000 0.0000	~

# Lets compare the data for level and first difference –



 Seem to show that the first difference is stationary – now we are going to test – ADF test – (also can apply ACF in this case – you can try it later)

### Lag selection-Before Cointegration

• Before performing cointegration test and VEC modelling, we need to determine the optimal number of lags

Anopeenedion	~ ~
Basics VAR Restrictions	
VAR type Standard VAR Vector Error Correction Bayesian VAR Estimation sample 1971q1 1991q4	Endogenous variables Igdp lpce lpdi lprofits Lag Intervals for Endogenous:
	Exogenous variables
	OK Cancel

VAR Specification

Vector Autoregression Estimates Date: 07/11/21 Time: 20:43 Sample (adjusted): 1971Q3 1991Q4 Included observations: 82 after adjustments Standard errors in () & t-statistics in []

	LGDP	LPCE	LPDI	LPROFITS
LGDP(-1)	0.619799	-0.098604	-0.088364	-2.498874
	(0.14932)	(0.13192)	(0.18546)	(1.29700)
	[ 4.15093]	[-0.74748]	[-0.47646]	[-1.92666]
LGDP(-2)	-0.058921	-0.265094	-0.077570	0.746310
	(0.14609)	(0.12907)	(0.18145)	(1.26898)
	[-0.40332]	[-2.05395]	[-0.42750]	[ 0.58812]
LPCE(-1)	0.642212	1.093103	0.826566	3.506485
	(0.15536)	(0.13726)	(0.19297)	(1.34953)
	[4.13363]	[7.96386]	[4.28342]	[2.59831]
LPCE(-2)	-0.045537	0.332415	-0.343753	-0.466922
	(0.18726)	(0.16544)	(0.23258)	(1.62657)
	[-0.24318]	[ 2.00933]	[-1.47797]	[-0.28706]
LPDI(-1)	0.007415	0.004783	0.575371	-0.176102
	(0.09614)	(0.08493)	(0.11941)	(0.83508)
	[ 0.07713]	[0.05631]	[4.81850]	[-0.21088]
LPDI(-2)	-0.237745	-0.118214	0.048937	-1.364373
	(0.09638)	(0.08515)	(0.11971)	(0.83719)
	[-2.46672]	[-1.38832]	[0.40879]	[-1.62970]
LPROFITS(-1)	0.005711	-0.027753	-0.056451	1.112872
	(0.01487)	(0.01314)	(0.01847)	(0.12916)
	[ 0.38410]	[-2.11259]	[-3.05653]	[8.61611]
LPROFITS(-2)	-0.003098	0.023917	0.059836	-0.163436

## Lag selection

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	2	961	.23	13	44.639	80*	4.	32e-16	*	-24.031	98* -	22.936	18	-23.5	9367*
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	4	982	.36	93	12.128	381	5.	87e-16		-23.749	85 -	21.680	00	-22.9	2193
	5	988	.84	70	9.4221	142	7.	72e-16	i.	-23.502	52 ·	20.945	64	-22.4	17979
	6	100	2.8	30	18.885	567	8.	48e-16		-23.450	12 ·	20.406	22	-22.2	23259
	7	101	3.0	49	12.740	066	1.	04e-15		-23.299	97 -	19.769	04	-21.8	38763

\* indicates lag order selected by the criterion

- LR: sequential modified LR test statistic (each test at 5% level)
- FPE: Final prediction error
- AIC: Akaike information criterion
- SC: Schwarz information criterion
- HQ: Hannan-Quinn information criterion

- Based on LR FPE AIC SC HQ, the lag selected is lag 1 and 2
- We use this for cointegration test

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		(0.18726	5) 51	(0.16544)	(0.2325	8) (1.626	557) 7061		
	, i	-0.24318	51	[2.00933]	[-1.4779	/] [-0.28/	00]		
LPDI(-1)	)	0.00741	5	0.004783	0.57537	1 -0.176	102		
		(0.09614	4)	(0.08493)	(0.1194	1) (0.835	508)		
	I	[0.07713	3]	[0.05631]	[ 4.8185	0] [-0.210	)88]		
LPDI(-2)	) -	-0.23774	5	-0.118214	0.04893	-1.364	373		
		(0.09638	3)	(0.08515)	(0.1197	1) (0.837	719)		
	I	-2.46672	2]	[-1.38832]	[ 0.4087	9] [-1.629	970]		
L PROFITS	(-1)	0.00571	1	-0.027753	-0.05645	51 1 1 1 2	872		
Entorno		(0.01487	ò	(0.01314)	(0.0184	7) (0.129	916)		
	I	0.38410	j	[-2.11259]	[-3.0565	3] [8.616	511 <u>j</u>		
	(-2)	0 00300	Q	0.023017	0.05083	-0.163	436		
LINOFIIO	( - )	0.000000		0.020011	0.05905	-0.103	400		

#### 10

### Cointegration

- After verifying variables are I(1), we run Johansen Cointegration Test
- The LAGS determined by lag selections criteria (here, 1 2 or 2 lags

Johannan Cointegration Test			$\sim 1$	EViews - [Var: l	UNTITLED Workf	file: TIME SERIES	PART 1::Untitled\]	
ionansen Cointegration Test			$\sim 1$	Can File Edit O	bject View Pr	oc Quick Op	tions Add-ins W	Vindow Help
Cointegration Test Specification	VEC Restrictions			View Proc Object	Print Name F	reeze Estimate	Forecast Stats Imp	pulse Resids
Deterministic trend assumption Assume no deterministic trend () 1) No intercept or trend	n of test I in data: in CE or test VAR	Exog variables*		Date: 07/11/21 Sample (adjuste Included observa Trend assumptio Series: LGDP LP Lags interval (in t	Time: 20:48 d): 1971Q4 1991 ations: 81 after ac on: Linear detern CE LPDI LPROF first differences):	Q4 djustments ninistic trend iTS 1 to 2		
<ul> <li>2) Intercept (no trend) ir</li> </ul>	n CE - no intercept in VAR			Unrestricted Coir	ntegration Rank	Test (Trace)		
Allow for linear deterministic t	rend in data:	Lag intervals		Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
<ul> <li>③ 3) Intercept (no trend) in</li> <li>④ 4) Intercept and trend in</li> </ul>	n CE and test VAR n CE - no intercept in VAR	1 2 Lag spec for differenced		None * At most 1 At most 2 At most 3	0.389571 0.136256 0.096042 0.024647	62.04596 22.06495 10.20019 2.021403	47.85613 29.79707 15.49471 3.841466	0.0014 0.2949 0.2657 0.1551
Allow for quadratic determinis 5) Intercept and trend in	itic trend in data: i CE - intercept in VAR	endogenous Critical Values		Trace test indica * denotes rejecti **MacKinnon-Ha	tes 1 cointegrati on of the hypothe aug-Michelis (199	ng eqn(s) at the esis at the 0.05 99) p-values	0.05 level level	
	<b>c</b>			Unrestricted Coir	ntegration Rank	Test (Maximum	Eigenvalue)	
() 6) Summarize all 5 sets o	r assumptions	Size 0.05		Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
* Critical values may not be va variables; do not include C or T	lid with exogenous Trend.	Osterwald-Lenum		None * At most 1 At most 2 At most 3	0.389571 0.136256 0.096042 0.024647	39.98101 11.86476 8.178791 2.021403	27.58434 21.13162 14.26460 3.841466	0.0008 0.5611 0.3608 0.1551
		OK Cancel		Max-eigenvalue * denotes rejecti **MacKinnon-Ha	test indicates 1 o on of the hypothe aug-Michelis (199	cointegrating eq esis at the 0.05 99) p-values	n(s) at the 0.05 leve level	el
				Unrestricted Coi	ntegrating Coeffi	icients (normali:	zed by b'*S11*b=I):	

#### Vector Error Correction model

- If nonstationary but I(1) time series are cointegrated, we can run the VECM to examine both the short-run and long-run dynamics of the series
- Conventional ECM for cointegrated series:

$$\Delta y_t = \beta_0 + \sum_{i=1}^n \beta_i \Delta y_{t-i} + \sum_{i=0}^n \delta_i \Delta x_{t-i} + \phi z_{t-1} + \mu_t$$

• z is the ECT and is the OLS residuals from the following long-run cointegrating regression:

$$\mathbf{y}_{t} = \boldsymbol{\beta}_{0} + \boldsymbol{\beta}_{1}\mathbf{x}_{t} + \boldsymbol{\varepsilon}_{t}$$

...and is defined as

 $z_{t-1} = ECT_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1}$ 

- The term, *error-correction*, relates to the fact that last period deviation from long-run equilibrium (the *error*) influences the short-run dynamics of the dependent variable
- Thus, the coefficient of ECT, φ, is the speed of adjustment, because it measures the speed at which Y returns to equilibrium after a change in X.

Vector Error Correction model



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-0.002102

(0.01647)

[-0.12761]

Vector Error Correction Estimates Date: 07/11/21 Time: 20:52 Sample (adjusted): 1971Q4 1991Q4 Included observations: 81 after adjustments Standard errors in ( ) & t-statistics in []

D(LPROFITS(-2))

Cointegrating	Eq: C	ointEq1		
LGDP(-1)	1.	000000		
LPCE(-1)	-1 (0 [-9	.326357 0.13437) 0.87064]		
LPDI(-1)	0. (0 [ 3	.478409 ).14309) 3.34331]		
LPROFITS(-	1) 0. (0 [1	.012763 ).00782) ].63203]		
С	-1	.724477		
Error Correction:	D(LGDP)	D(LPCE)	D(LPDI)	D(LPROFITS)
CointEq1	-0.342926	-0.448602	-0.307187	-2.443886
	(0.08798)	(0.07148)	(0.10507)	(0.73924)
	[-3.89777]	[-6.27560]	[-2.92357]	[-3.30596]
D(LGDP(-1))	0.051226	0.343598	0.214043	0.318199
	(0.17755)	(0.14426)	(0.21204)	(1.49183)
	[ 0.28852]	[2.38183]	[ 1.00943]	[0.21329]
D(LGDP(-2))	0.137742	0.272724	0.588286	-0.760001
	(0.15382)	(0.12498)	(0.18370)	(1.29242)
	[ 0.89550]	[2.18221]	[ 3.20243]	[-0.58804]
D(LPCE(-1))	0.101643	-0.601986	0.187181	-0.410255
	(0.23268)	(0.18905)	(0.27789)	(1.95507)
	[ 0.43683]	[-3.18421]	[ 0.67359]	[-0.20984]
D(LPCE(-2))	-0.031397	-0.361252	-0.452834	-1.253149
	(0.20637)	(0.16768)	(0.24647)	(1.73403)
	[-0.15214]	[-2.15443]	[-1.83728]	[-0.72268]
D(LPDI(-1))	0.188508	0.222084	-0.105357	1.362502
	(0.11012)	(0.08947)	(0.13151)	(0.92527)
	[ 1.71183]	[2.48213]	[-0.80111]	[ 1.47254]
D(LPDI(-2))	-0.057598	0.120343	-0.141553	0.578921
	(0.10160)	(0.08255)	(0.12133)	(0.85365)
	[-0.56693]	[ 1.45788]	[-1.16664]	[0.67817]
D(LPROFITS(-1))	0.007016	-0.022089	-0.045967	0.166455
	(0.01505)	(0.01223)	(0.01798)	(0.12648)
	[ 0.46613]	[-1.80608]	[-2.55699]	[ 1.31609]

-0.020981

(0.01338)

[-1.56763]

-0.004498

[-0.22865]

(0.01967)

0.004291

(0.13841)

## The output

#### • Estimated VECM with LGDP as target variable:

•  $DLGDP_t = -0.34292582(ECT(-1)) + 0.051225999D(LGDP(-1)) + 0.137742438D(LGDP(-2)) + 0.101643257D(LPCE(-1)) - 0.031397295D(LPCE(-2)) + 0.188508359D(LPDI(-1)) + 0.057597656D(LPDI(-2)) + 0.007016434 D(LPROFITS(-1)) - 0.00210206 D(LPROFITS(-2)) + 0.007016434 D(LPROFITS(-2)) - 0.007016434 D(LPROFIT$ 

0.003549

#### **Cointegrating equation (long-run model):**

• Ect(-1) = 1.000 LGDP(-1) -1.32636 LPCE(-1) +0.478409 LPDI(-1) +0.012763477 LPROFITS(-1) -1.72448

#### Make into a system to estimate – finding P value

(-2)) + C(8)*D(LPROFITS(-1)) + C(9)*D(LPROFITS(-2)) + C(10)		
D(LPCE) = C(11)*(LGDP(-1) - 1.32635670644*LPCE(-1) + 0.478409375697*LPDI(-1) + 0.01276347674 1.72447733078) + C(12)*D(LGDP(-1)) + C(13)*D(LGDP(-2)) + C(14)*D(LPCE(-1)) + C(15)*D(LPCE(-2)) + *D(LPDI(-2)) + C(18)*D(LPROFITS(-1)) + C(19)*D(LPROFITS(-2)) + C(20) D(LPDI) = C(21)*(LGDP(-1) - 1.32635670644*LPCE(-1) + 0.478409375697*LPDI(-1) + 0.012763476749 1.72447733078) + C(22)*D(LGDP(-1)) + C(23)*D(LGDP(-2)) + C(24)*D(LPCE(-1)) + C(25)*D(LPCE(-2)) + *D(LPDI(-2)) + C(28)*D(LPROFITS(-1)) + C(29)*D(LPROFITS(-2)) + C(30) D(LPROFITS) = C(31)*(LGDP(-1) - 1.32635670644*LPCE(-1) + 0.478409375697*LPDI(-1) + 0.01276347 1.72447733078) + C(32)*D(LGDP(-1)) + C(33)*D(LGDP(-2)) + C(34)*D(LPCE(-1)) + C(35)*D(LPCE(-2)) + *D(LPDI(-2)) + C(38)*D(LPROFITS(-1)) + C(39)*D(LPROFITS(-2)) + C(40)	ar: VAR01 Workfile: TIME SERIES PART 1::Un Proc Object Print Name Freeze Estima Specify/Estimate Make Residuals Make Structural Residuals Make Model Make Endogenous Group Make Cointegration Group Make System Estimate Structural Factorization Add-ins [-9.87064] LPDI(-1) 0.478409 (0.14309) [3.34331] LPROFITS(-1) 0.012763 (0.00782) [1.63203] C -1.724477	titled\ te Forecast Stats Impulse Resids imates Order by Variable Order by Lag

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	ĩ	and PDL terms, OR an explicit eq	uation like Y=c(	1)+c(2)*X.	•
	D(LGDP)	= C(1)*(LGDP(-1) - 1.3263567	0644*LPCE(-1)	+ 0.478409375	697*LPDI
	(-1) + 0	.0127634767491*LPROFITS(-1)	- 1.724477330	78) + C(2)*D(L	GDP(-1))
	+ C(3)*	D(LGDP(-2)) + C(4)*D(LPCE(-1)) D(LPDI(-2)) + C(8)*D(LPCE(-1))	) + C(5)*D(LPCE (-1)) + C(9)*D(	:(-2)) + C(6)*D( ) PROFITS(-2))	(LPDI(-1)) + C(10)
			( 1)) - 0(0) 0(	2.110.110(2))	
	Estimatio	n settings			
	Method:	LS - Least Squares (NLS and A	RMA)		~
	Sampler				
	bampie.	1971Q1 1991Q4			
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Finding A p Value for the error correction term – abut 34 percent departure from long run equilibrium corrected each period – the independent variable granger causes LGDP in the long run

#### EViews - [Equation: UNTITLED Workfile: TIME SERIES PART 1::Untitled\]

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\begin{array}{l} \mbox{Dependent Variable: D(LGDP)} \\ \mbox{Method: Least Squares (Gauss-Newton / Marquardt steps)} \\ \mbox{Date: 07/11/21 Time: 21:09} \\ \mbox{Sample (adjusted): 1971Q4 1991Q4} \\ \mbox{Included observations: 81 after adjustments} \\ \mbox{D(LGDP) = C(1)*( LGDP(-1) - 1.32635670644*LPCE(-1) + 0.478409375697 \\ *LPDI(-1) + 0.0127634767491*LPROFITS(-1) - 1.72447733078 ) + C(2) \\ *D(LGDP(-1)) + C(3)*D(LGDP(-2)) + C(4)*D(LPCE(-1)) + C(5)*D(LPCE(-2)) + C(6)*D(LPDI(-1)) + C(7)*D(LPDI(-2)) + C(8)*D(LPROFITS(-1)) + \\ C(9)*D(LPROFITS(-2)) + C(10) \\ \end{array}
```

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.342926	0.087980	-3.897768	0.0002
C(2)	0.051226	0.177549	0.288517	0.7738
C(3)	0.137742	0.153817	0.895497	0.3735
C(4)	0.101643	0.232682	0.436833	0.6636
C(5)	-0.031397	0.206375	-0.152137	0.8795
C(6)	0.188508	0.110121	1.711832	0.0913
C(7)	-0.057598	0.101596	-0.566927	0.5726
C(8)	0.007016	0.015053	0.466127	0.6426
C(9)	-0.002102	0.016473	-0.127607	0.8988
C(10)	0.003549	0.001617	2.195118	0.0314
R-squared	0.480903	Mean depend	lent var	0.006117
Adjusted R-squared	0.415101	S.D. depende	ent var	0.009836
S.E. of regression	0.007522	Akaike info cr	iterion	-6.826747
Sum squared resid	0.004018	Schwarz crite	rion	-6.531136
Log likelihood	286.4833	Hannan-Quin	in criter.	-6.708144
F-statistic	7.308428	Durbin-Watso	on stat	1.975212
Prob(F-statistic)	0.000000			

#### Causality for the short run variables

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	Grad	lients a	and Deriv	atives		356706	44*LP	PCE(-1)	+ 0.478	40937569	17
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		inarree	machix		_ [	)(LPDI(	2))+	C(8)*D	(LPROF	TTS(-1)) +	-
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		C(4)		0.1	0164						
		C(5)		-0.0	3139	Wa	ld Tes	t- Coeff	icient Re	estrictions	•
		C(6)		0.1	8850	Om	itted	Variable	es Test - L	likelihood	Ratio
		C(2)		-0.0	0701	Rei	lunda	nt Varia	hles Test	t - Likeliho	od Ratio
		C(9)		-0.0	0210	-			ibles les	C - LIKCIIIIO	
		C(10	)	0.0	0354	Fac	tor Br	eakpoir	nt Test		
_		od			0000	2 1400	n dar	andan	tuer	0.0064	

_	
Va	ald Test
	Coefficient restrictions separated by commas
	C(4)=C(5)=C(6)=C(7)=C(8)=C(9)=0

- Does all independent variables granger cause LGDP?
- No causality between LGDP, LPCE,LPDI and LPROFITS or no short run relationship

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/iew	Proc	Objec	ct 🗍	Print	Name	Freeze	e 🗍	Estim	nate]	Foreca	st]	Stats	Resids

#### Wald Test:

Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	1.006736	(6, 71)	0.4279
Chi-square	6.040415	6	0.4187

Null Hypothesis: C(4)=C(5)=C(6)=C(7)=C(8)=C(9)=0 Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	0.101643	0.232682
C(5)	-0.031397	0.206375
C(6)	0.188508	0.110121
C(7)	-0.057598	0.101596
C(8)	0.007016	0.015053
C(9)	-0.002102	0.016473

Restrictions are linear in coefficients.

### SERIAL CORRELATION

#### P value is > than α so no serial correlation

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Null hypothesis: No seria	al correlation a	at up to 2 lags		
F-statistic	1.232481	Prob. F(2,69)		0.2979
Obs*R-squared	2,793843	Prob. Chi-Sau	are(2)	0.2474
Dependent Variable: RES	SID			
Method: Least Squares	50			
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Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.135680	0.278611	-0.486989	0.6278
C(2)	-0.649616	0.607335	-1.069616	0.2885
C(3)	0.257058	0.224309	1.145996	0.2558
C(4)	-0.207758	0.422036	-0.492275	0.6241
C(5)	0.268153	0.267758	1.001476	0.3201
C(6)	0.082097	0.191449	0.428820	0.6694
C(7)	0.105485	0.185171	0.569661	0.5708
C(8)	0.000595	0.015013	0.039601	0.9685
C(9)	0.004189	0.017681	0.236918	0.8134
C(10)	0.000685	0.003170	0.215997	0.8296
RESID(-1)	0.806569	0.841377	0.958630	0.3411
RESID(-2)	-0.435144	0.293022	-1.485025	0.1421
R-squared	0.034492	Mean dependent var		-7.98E-18
Adjusted R-squared	-0.119430	S.D. dependent var		0.007087
S.E. of regression	0.007498	Akaike info criterion		-6.812465
Sum squared resid	0.003879	Schwarz criterion		-6.457732
Log likelihood	287.9048	Hannan-Quinn criter.		-6.670141
F-statistic	0.224087	Durbin-Watson stat		2.039448
Prob(F-statistic)	0.995219			



# Key Concepts

- 1. Stochastic Processes
  - i. Stationarity Processes
  - ii. Purely Random Processes
  - iii. Non-stationary Processes
- 2. Random Walk Models
  - i. Random Walk with Drift
  - ii. Random Walk without Drift
- 3. Unit Root Stochostic Processes
- 4. Deterministic and Stochastic Trends
- 5. The Phenomenon of Spurious Regression
- 6.Tests of Stationarity/non-stationarity
  - i. Graphical Method
  - ii. Unit Root Tests