



# Structural Equation Modeling

# Structural Equation Modelling

- A so-called *second-generation data analysis method*
  - First generation data analysis methods include techniques such as **regression**, (multivariate) analysis of (co-)variance (ANOVA).
  - They are characterized by their shared limitation of being able to analyse only one layer of linkages between independent and dependent variables at a time.
  - 2<sup>nd</sup> generation methods allow for the simultaneous analysis of multiple independent and dependent variables
- encourages confirmatory rather than exploratory analysis.

Uni-variate Analysis			Bi-variate Analysis		
			Dependent Variable		
1 variable				Metric	Non-Metric
Metric	Mean, Median, Sigma, 1-sample t-test	Independent Variable	Metric	Correlation	Linear Discriminant Analysis
	<b>Descriptive Statistics-&gt; Descriptive</b>			<b>Correlation-&gt;Bivariate</b>	<b>Classify-&gt;Discriminant</b>
Non-Metric	Count, Percent	Variable	Non-Metric	Simple Regression	Binary Regression
	<b>Descriptive Statistics-&gt; Frequency</b>			<b>Regression-&gt;Linear</b>	<b>Regression-&gt;Binary Logistics</b>
				Hypo testing	Cross-tab / Chi sq test
				<b>Compare Means</b>	<b>Descriptive Statistics-&gt; Crosstab</b>

E16 Scale

	B	C	D	E	F	G	H	I	J	K
12					Dependent Variable					
13					Ratio		Scale			
14				Ratio	Simple Regression		SEM=CFA+Path Analysis			
15				Scale	SEM=CFA+Path Analysis		SEM=CFA+Path Analysis			

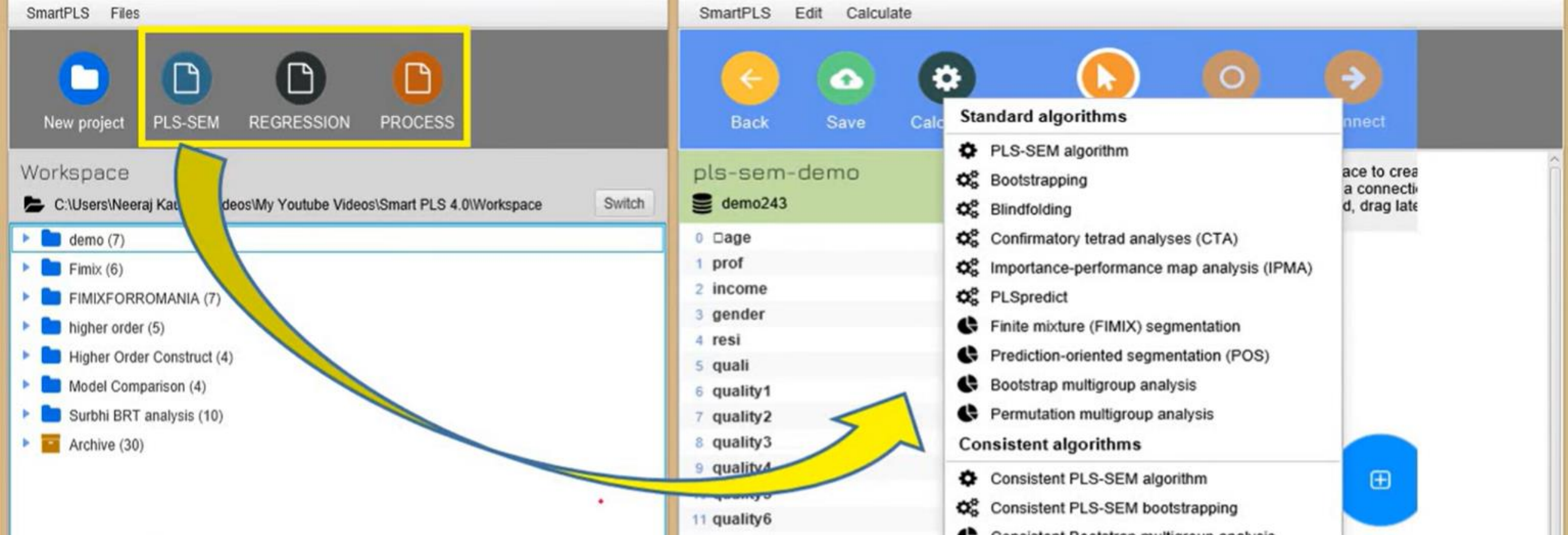
SEM= Measurement Model (CFA) + Structural Model (Path Analysis or Regression)

	CFA	Path Analysis
21	Reliability	Beta
22	Validity	r <sup>2</sup> / Adj r <sup>2</sup>
23	Model fitness	p-value
24		Mediation
25		Moderation

SmartPLS

A	B	C	D	E	F	G	H	I
				SEM= Measurement Model (CFA) + Structural Model (Path Analysis)				
				<b>CFA</b>		<b>Path Analysis</b>		
				Reliability		Beta		
				Validity		r2 / Adj r2		
				Model fitness		p-value		
						Mediation		
						Moderation		
				<b>Algorithm</b>				
				MLE	AMOS			
				PLS	SmartPLS			
				OLS	SPSS/Excel			





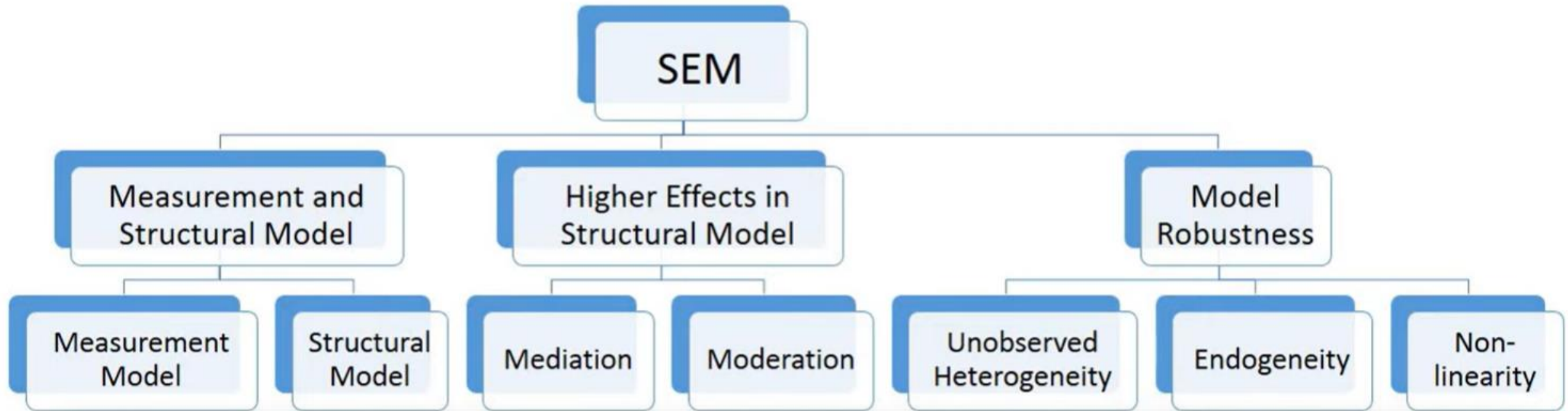
## Introduction to various analyses available in SmartPLS v4



## Navigation

- Indicators
- Indicator correlations
- Data groups
- Raw data

Name	Mean	Median	Scale min	Scale max	Observed min	Observed max	Standard deviation	Excess kurtosis	Skewness	Cramér-von Mises p value
age	1.831	2.000	1.000	3.000	1.000	3.000	0.661	-0.744	0.199	0.000
prof	2.247	2.000	1.000	4.000	1.000	4.000	0.963	-0.489	0.657	0.000
income	2.004	2.000	1.000	3.000	1.000	3.000	0.849	-1.620	-0.008	0.000
gender	1.288	1.000	1.000	2.000	1.000	2.000	0.453	-1.122	0.942	0.000
resi	1.082	1.000	1.000	2.000	1.000	2.000	0.275	7.416	3.059	0.000
quali	2.457	2.000	1.000	4.000	1.000	4.000	1.093	-1.266	0.273	0.000
quality1	3.745	4.000	1.000	5.000	1.000	5.000	0.982	-0.132	-0.546	0.000
quality2	3.852	4.000	1.000	5.000	1.000	5.000	0.974	0.353	-0.773	0.000
quality3	3.638	4.000	1.000	5.000	1.000	5.000	0.952	0.173	-0.515	0.000
quality4	3.494	4.000	1.000	5.000	1.000	5.000	1.059	-0.034	-0.631	0.000
quality5	3.827	4.000	1.000	5.000	1.000	5.000	0.940	0.804	-0.844	0.000
quality6	3.626	4.000	1.000	5.000	1.000	5.000	0.905	0.498	-0.496	0.000





# When do we use SEM?

- Complex research models:
  - Multiple associations between multiple independent and multiple dependent variables
  - Usually also mediating and/or moderating variables present
- Latent concepts: Multi-dimensional constructs with several underlying dimensions
  - Satisfaction, usefulness, attitude etc.
  - Constructs that have multiple measures
  - Often measured with perceptual (self-report) data
- Often: survey research but also in experiments and others

# Latent constructs

- abstractions about a phenomenon (e.g. usefulness, time, satisfaction, enjoyment) that are **latent** in that they relate to a real thing but do not have a tangible existence:
  - Thus, they cannot be measured directly
- have indicators associated with them:
  - **Measures** are our approximations to latent constructs – our empirical indicators that allow us to ‘grasp’ the latent construct.
  - 1+ measure required per construct dimension (also called substratum)
  - Typically multiple items because most constructs are indeed complex concepts that have multiple domains of meaning.



# SEM Overview

## Four phases of analysis

1. (Descriptive statistics)
2. Measurement model estimation
3. Structural model estimation
4. Mediation/Moderation/supplementary analyses

# Descriptive Statistics

e.g., assessment of non-response error

- Chi-square test of early versus late survey respondents

Demographic variable	p-value
Type of organisation	.206
Size of organisation	.436
Size of modelling team	.305
Country of origin	.100
Years of experience in process modelling overall	.346
Months of experience in process modelling with BPMN	.639
Number of BPMN models created	.345
Type of training	.784
Use of modelling tool	.060
Use of modelling guidelines	.311
Use of BPMN constructs	.542

# Structural Model Estimation: 5-stage process

- **Model specification**
  - Specification of an a-priori research model with theoretical constructs and hypothesized relationships between them.
- **Model identification**
  - Estimation of unknown parameters (such as factor loadings, path coefficients or explained variance) based on observed correlations or covariances.
- **Model estimation**
  - Finding of one set of model parameters that best fits the data.
- **Model fit testing**
  - Assessment of how well a model fits the data.
- **Model re-specification**
  - Improvement of either model parsimony or fit.



# Measurement Model Estimation

- Examines whether the model of what we measure fits the properties of the data we collected
- Often confused with confirmatory factor analysis.
- The actual test criteria (for reflective models) is Goodness of Fit.

Goodness of fit statistics for the measurement model (GFI = 0.91, NFI = 0.97, NNFI = 0.98, CFI = 0.98, SRMR = 0.05, RMSEA = 0.06,  $\chi^2 = 436.71$ , df = 155) suggest good fit of the measurement model to the data set, considering the approximate benchmarks suggested by Im and Grover (2004).

# Measurement Model Estimation for Reflective Measures

- Assessment of the reliability and validity of the scales used.
- Tests
  - Uni-dimensionality
    - A construct is uni-dimensional if its constituent items represent one underlying trait
  - Reliability and composite reliability
    - Reliability is defined as the degree to which scale items are free from error and, therefore, yield consistent results.
  - Convergent validity
    - Convergent validity tests if measures that should be related are in fact related.
  - Discriminant validity
    - Discriminant validity refers to the degree to which items of different constructs are unique from each other.

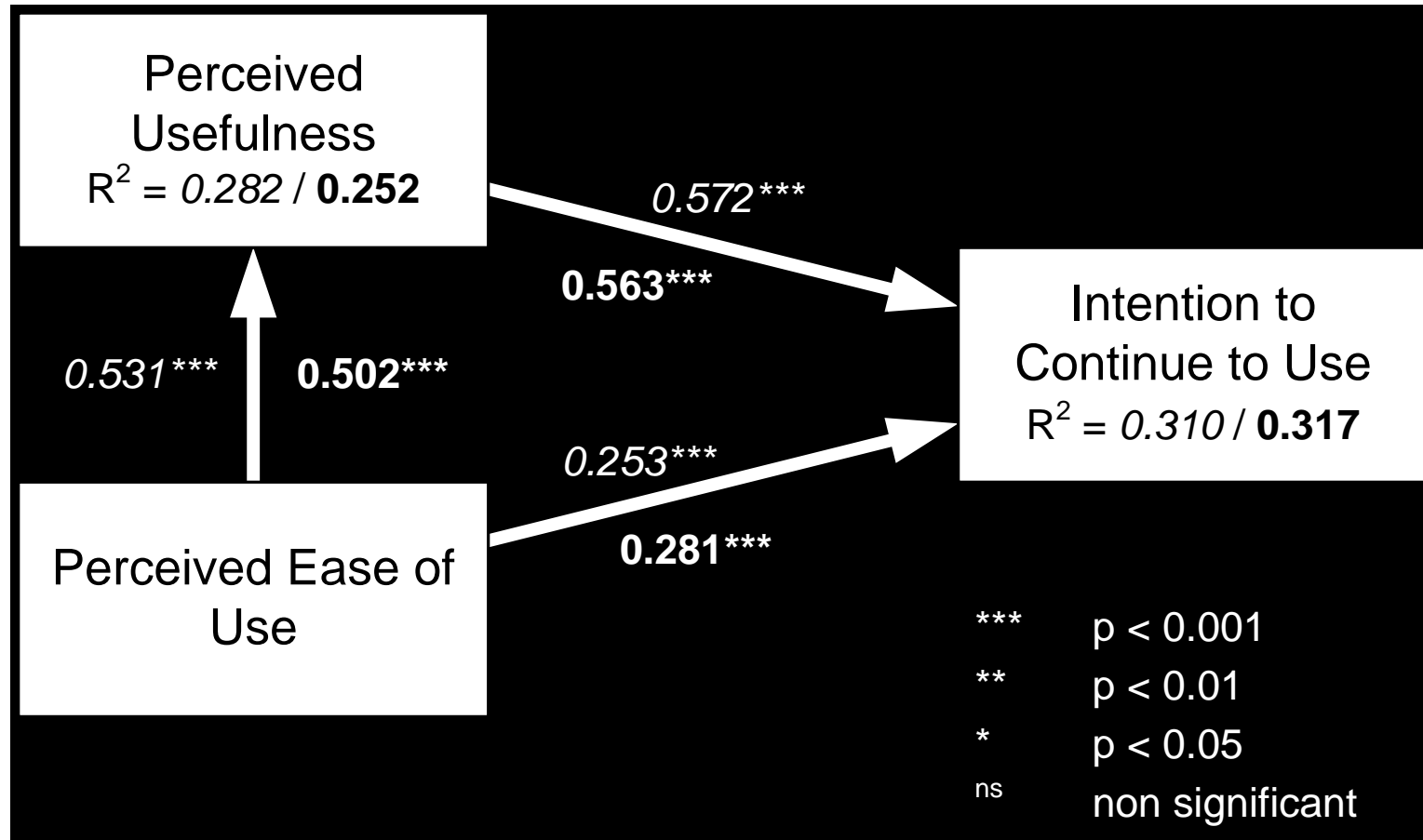
# Measurement Model Estimation

- Validation via standard set of indices (e.g., Fornell and Larcker, 1981)
  - Uni-dimensionality:
    - Cronbach's Alpha ( $\alpha$ ) > 0.7
  - Reliability:
    - Cronbach's Alpha ( $\alpha$ ) > 0.8
    - Composite reliability ( $\rho_c$ ) > 0.5
  - Convergent validity:
    - Average variance extracted (AVE) > 0.5
    - Indicator factor loadings ( $\lambda$ ) > 0.6
    - Indicator factor loadings significant at  $p < 0.05$
    - Composite reliability ( $\rho_c$ ) > 0.8
  - Discriminant validity:
    - AVE should exceed the squared correlations between each of the constructs

# Model Fit

Fit index	Suggested value	TAM (EPC)	ECT (EPC)	Hybrid (EPC)		TAM (BPMN)	ECT (BPMN)	Hybrid (BPMN)
GFI	> 0.900	0.942	0.932	0.926		0.956	0.950	0.934
AGFI	> 0.900	0.933	0.913	0.901		0.918	0.920	0.902
NFI	> 0.900	0.956	0.932	0.915		0.982	0.986	0.982
NNFI	> 0.900	0.946	0.923	0.905		0.979	0.986	0.985
CFI	> 0.900	0.964	0.943	0.927		0.986	0.990	0.988
SRMR	< 0.050	0.0439	0.0489	0.0496		0.0466	0.0433	0.0471
RMSEA	< 0.080	0.0731	0.0742	0.0784		0.0831	0.0693	0.070
$\chi^2 (df, p)$	-	119.383 (24, 0.000)	292.705 (49, 0.000)	537.519 (81, 0.000)		119.863 (24, 0.000)	190.000 (49, 0.000)	307.129 (81, 0.000)
R <sup>2</sup> for ItU	-	0.310	0.151	0.355		0.317	0.269	0.396

# Structural Model Estimation: Results Reporting





<b>Criteria</b>	<b>Variance-Based Modeling (e.g. SmartPLS, PLS Graph)</b>	<b>Covariance-Based Modeling (e.g. LISREL, AMOS, Mplus)</b>
<b>Objective</b>	Prediction oriented	Parameter oriented
<b>Distribution Assumptions</b>	Non-parametric	Normal distribution (parametric)
<b>Required sample size</b>	Small (min. 30 – 100)	High (min. 100 – 800)
<b>Model complexity</b>	Large models OK	Large models problematic (50+ indicator variables)
<b>Parameter Estimates</b>	Potential Bias	Stable, if assumptions met
<b>Indicators per construct</b>	One – two OK Large number OK	Typically 3 – 4 minimum to meet identification requirements
<b>Statistical tests for parameter estimates</b>	Inference requires Jackknifing or Bootstrapping	Assumptions must be met
<b>Measurement Model</b>	Formative and Reflective indicators OK	Typically only Reflective indicators
<b>Goodness-of-fit measures</b>	None	Many

## 2. Formative vs reflective models

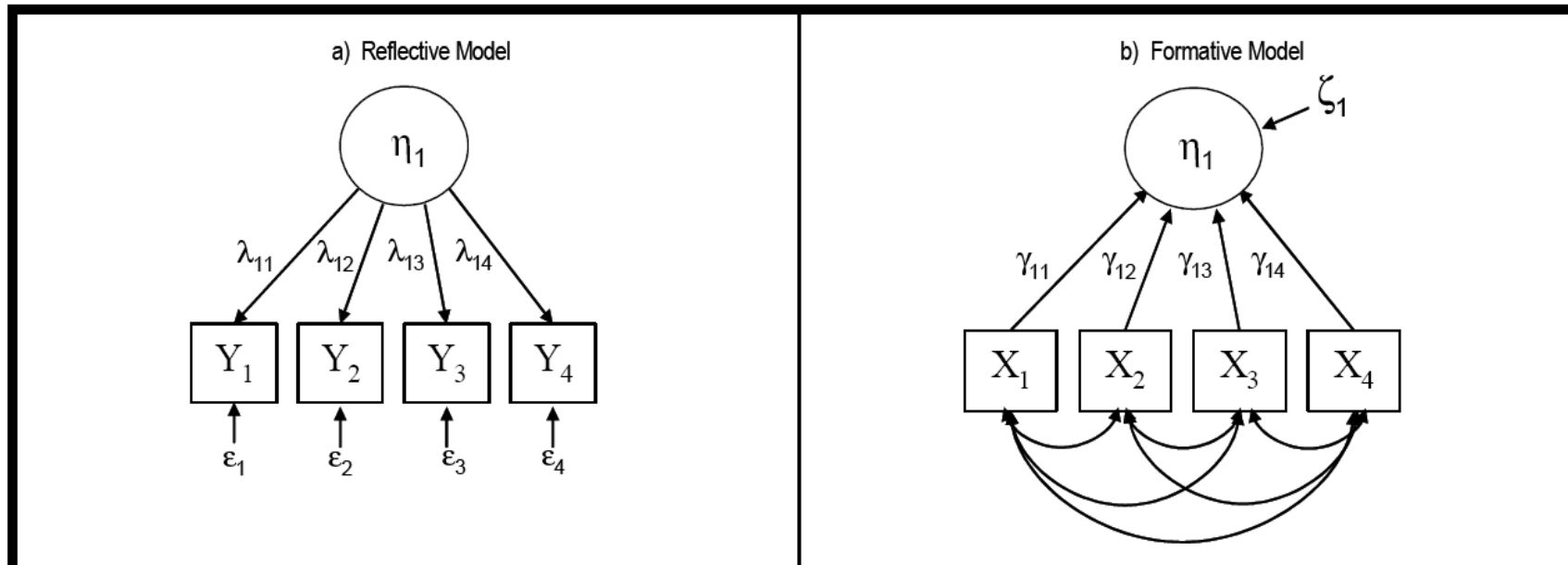
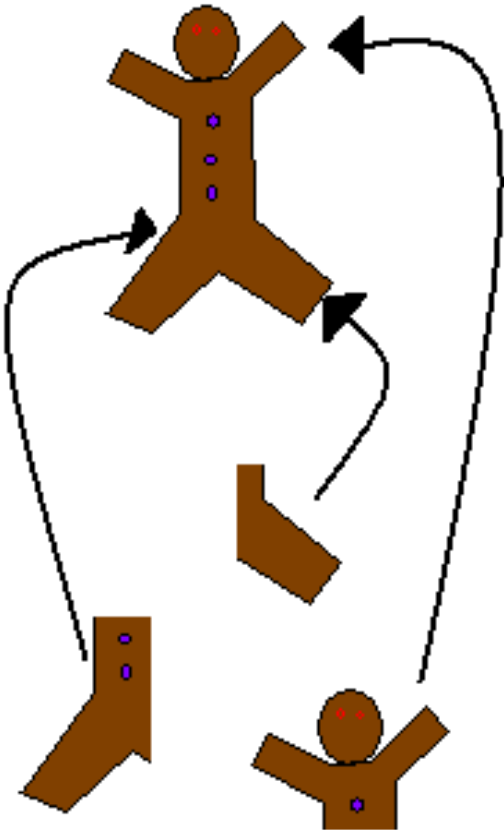


Figure 1. Diagram of Reflective and Formative Measurement Models (From K. Bollen and R. Lennox, "Conventional Wisdom on Measurement: A Structural Equation Perspective," *Psychological Bulletin* (110:2), 1991, pp. 305-314. Copyright © 1991 by the American Psychological Association. Reproduced with permission.)

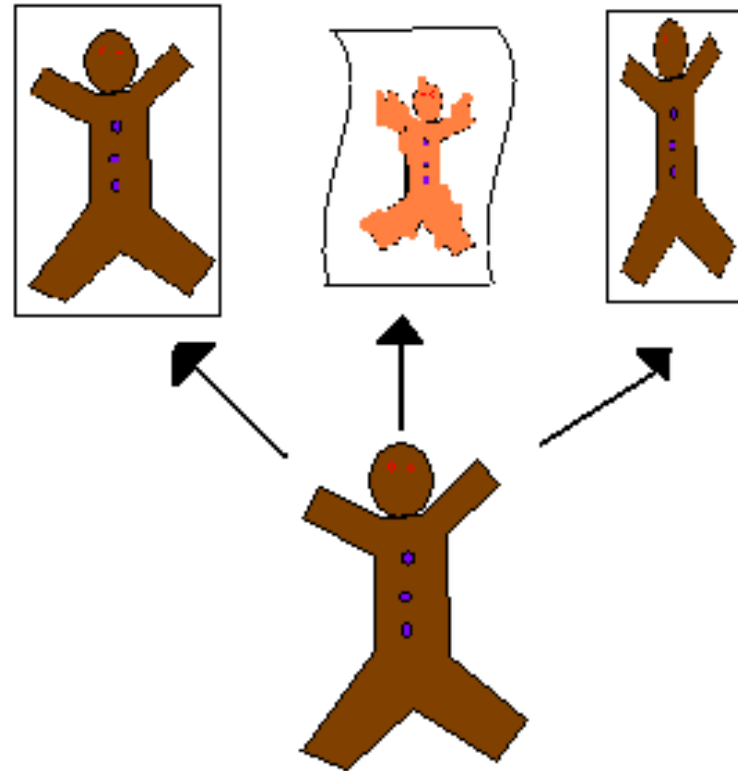
A construct could be measured reflectively or formatively.  
Constructs are not necessarily (inherently) reflective or formative.

# Formative vs reflective: Illustration

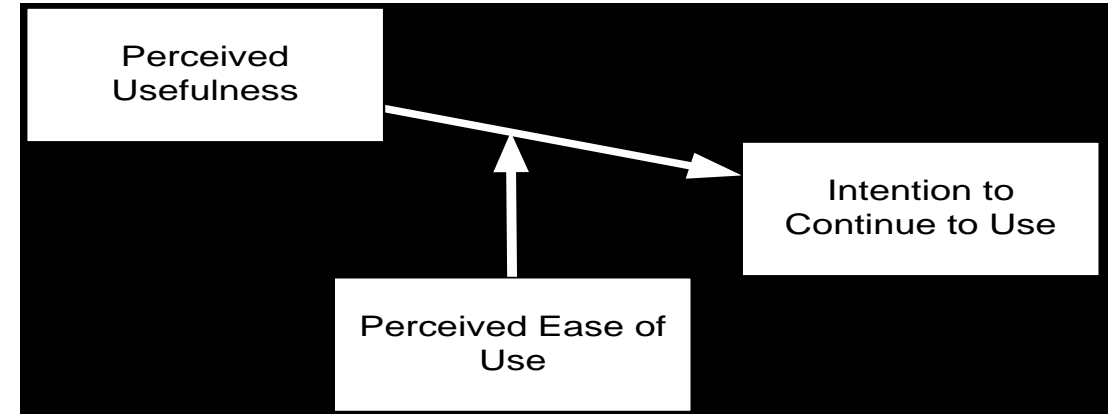
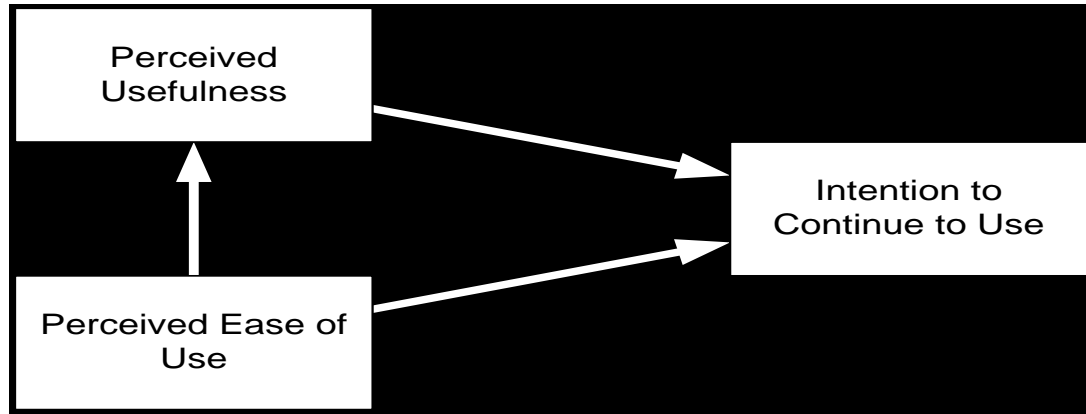
Formative Construct



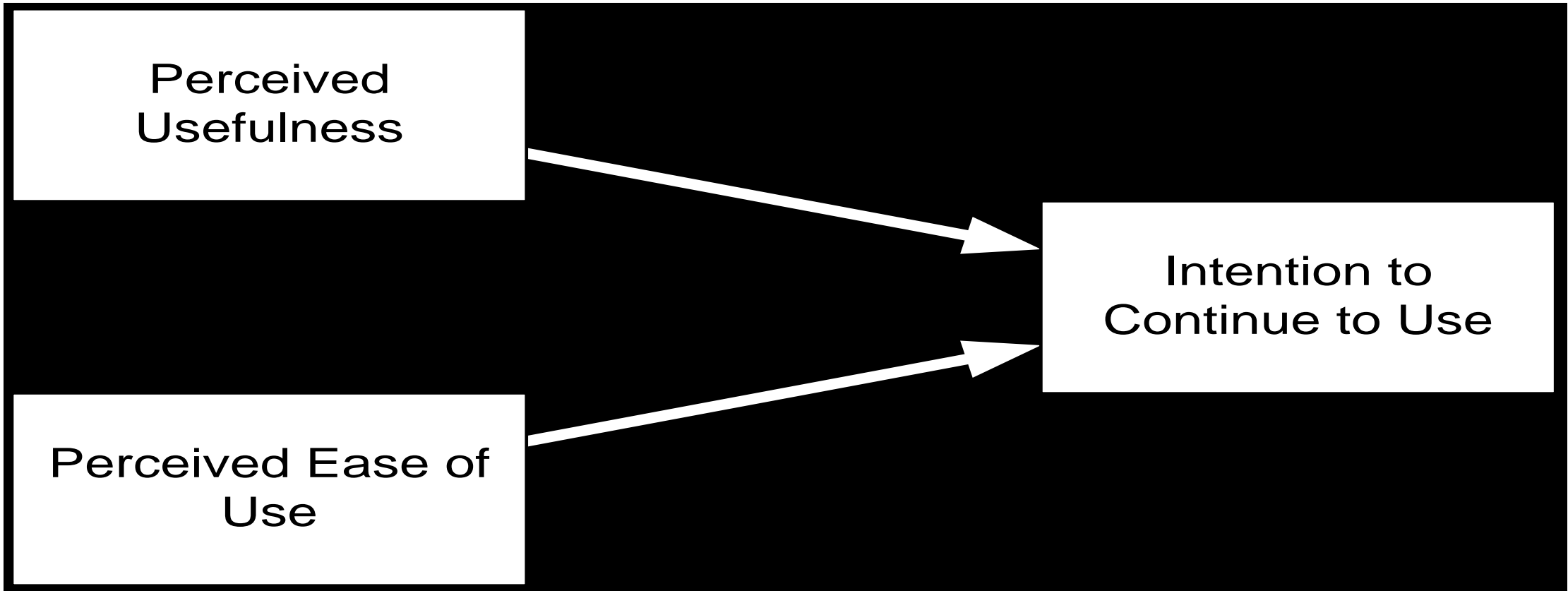
Reflective Construct



# 3. Mediation vs Moderation

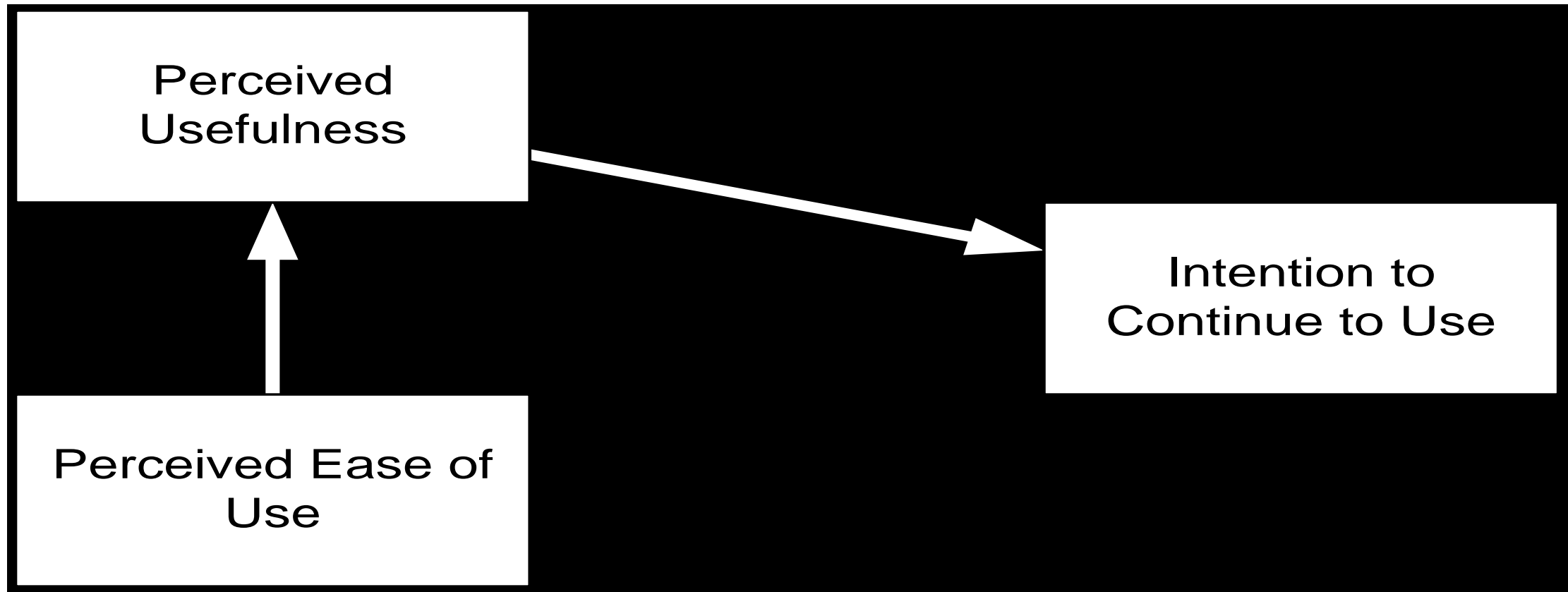


# No Mediation

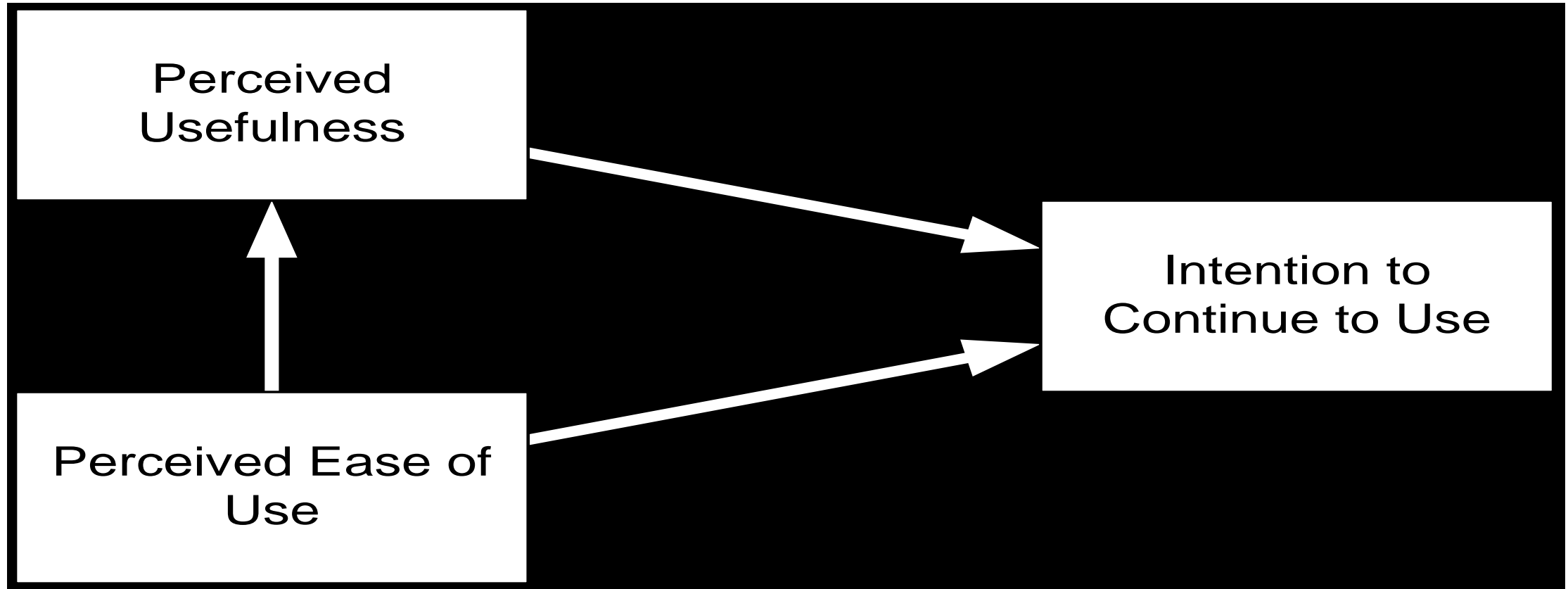




# Full Mediation



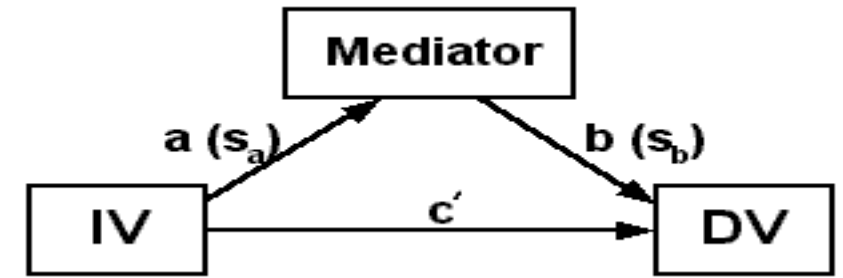
# Partial Mediation



# Sobel Mediation Test

- Numbers needed

- $a$  = raw (unstandardized) regression coefficient for the association between IV and mediator.
- $s_a$  = standard error of  $a$ .
- $b$  = raw coefficient for the association between the mediator and the DV (when the IV is also a predictor of the DV).
- $s_b$  = standard error of  $b$ .



- The Sobel test works well *only in large samples*. A better example includes bootstrapping of raw data: Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple Mediation models. *Behavior Research Methods, Instruments, & Computers*, 36, 717-731.

Sobel, M. E. (1982). Asymptotic intervals for indirect effects in structural equations models. In S. Leinhardt (Ed.), *Sociological methodology 1982* (pp.290-312). San Francisco: Jossey-Bass.

## CALCULATION FOR THE SOBEL TEST

*An interactive calculation tool for Mediation tests*

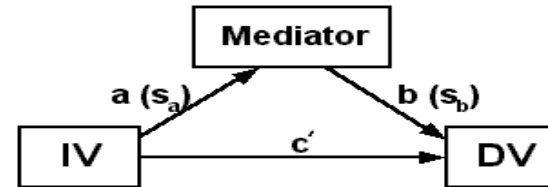
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given independent variable (IV) to a given dependent variable (DV). Generally speaking, Mediation can be said to occur when (1) the IV significantly affects the mediator, (2) the IV significantly affects the DV in the absence of the mediator, (3) the mediator has a significant unique effect on the DV, and (4) the effect of the IV on the DV shrinks upon the addition of the mediator to the model. These criteria can be used to informally judge whether or not Mediation is occurring, but MacKinnon & Dwyer (1993) and MacKinnon, Warsi, & Dwyer (1995) have popularized statistically based methods by which Mediation may be formally assessed.

### An illustration of Mediation

$a$ ,  $b$ , and  $c'$  are path coefficients. Values in parentheses are standard errors of those path coefficients.



### Description of numbers needed

$a$  = raw (unstandardized) regression coefficient for the association between IV and mediator.

$s_a$  = standard error of  $a$ .

$b$  = raw coefficient for the association between the mediator and the DV (when the IV is also a predictor of the DV).

$s_b$  = standard error of  $b$ .

### To get numbers

1. Run a regression analysis with the IV predicting the mediator. This will give  $a$  and  $s_a$ .
2. Run a regression analysis with the IV and mediator predicting the DV. This will give  $b$  and  $s_b$ . Note that  $s_a$  and  $s_b$  should never be negative.

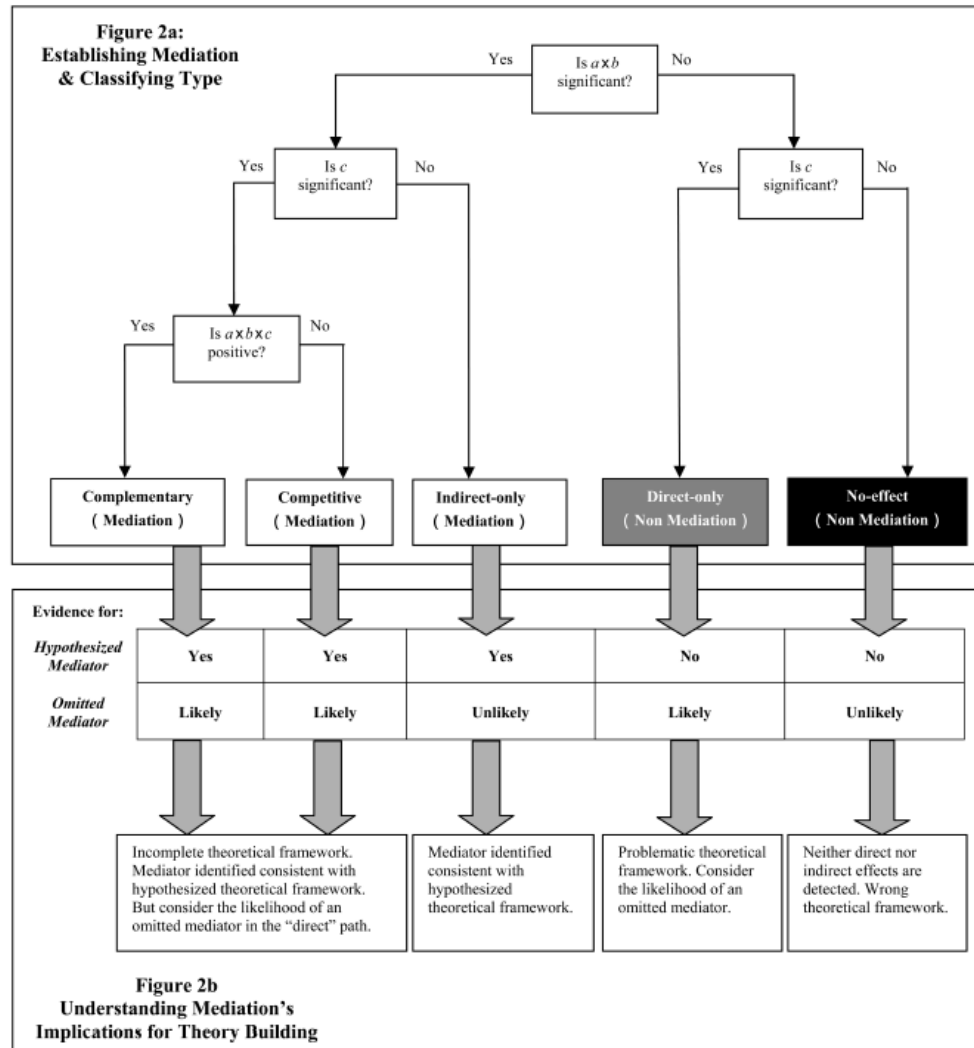
### To conduct the Sobel test

Details can be found in Baron and Kenny (1986), Sobel (1982), Goodman (1960), and MacKinnon, Warsi, and Dwyer (1995). Insert the  $a$ ,  $b$ ,  $s_a$ , and  $s_b$  into the cells below and this program will calculate the critical ratio as a test of whether the indirect effect of the IV on the DV via the mediator is significantly different from zero.

Input:	Test statistic:	Std. Error:	p-value:
$a$	Sobel test:		
$b$	Aroian test:		
$s_a$	Goodman test:		
$s_b$	<input type="button" value="Reset all"/>	<input type="button" value="Calculate"/>	

Alternatively, you can insert  $t_a$  and  $t_b$  into the cells below, where  $t_a$  and  $t_b$  are the  $t$ -test statistics for the difference between the  $a$  and  $b$  coefficients and zero. Results should be

# Procedure: Zhao et al. (2010)



Zhao, X., Lynch Jr., J.G., and Chen, Q. "Reconsidering Baron and Kenny: Myths and Truths about Mediation Analysis," *The Journal of Consumer Research* (37:2) 2010, pp 197-206.

# Moderation

